3. Basic principles of solar geometry

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Learning Outcomes

After studying this chapter, readers will be able to:

- understand the sun-earth geometry and the basic solar geometry angles
- perform various calculations between solar angles
- distinguish the difference between solar and local time
- define the position of the sun in relation to the earth
- explain the diurnal and annual apparent path of the sun's motion
- calculate the solar incidence angle for a plane oriented and tilted arbitrarily relative to the earth

This chapter describes the sun-earth geometry and how it can be used to define the position of the sun in relation to the earth. For the solar energy applications, it is important to understand the apparent motion of the sun, as well as the sun-earth angles. Understanding this mechanism will be of importance in the following chapters, which include elementary notions of astronomy. Thus, the emphasis is on the notions and terms that concern the sun-earth angles and their interrelationships, which are of high importance in solar geometry.

3.1 The Earth's rotation

The earth rotates around its own axis, known as the *Polar Axis* P P' (Fig. 3.1). The points at which this axis intercepts the earth are the north (N_p) and the south (S_p) poles. The great circle EIWI' normal to this axis is called equator and the plane containing the equator is the equatorial plane that divides the earth in northern and southern hemisphere. The great circle ESWN normal to the axis ZZ' is called horizon.
The position of the sun in the sky varies throughout the day and season due to the rotation of the earth around its axis once per day. Similarly, it changes its elliptical orbit around the sun, once per year, with the sun at one focus of the ellipse (Fig. 3.2).

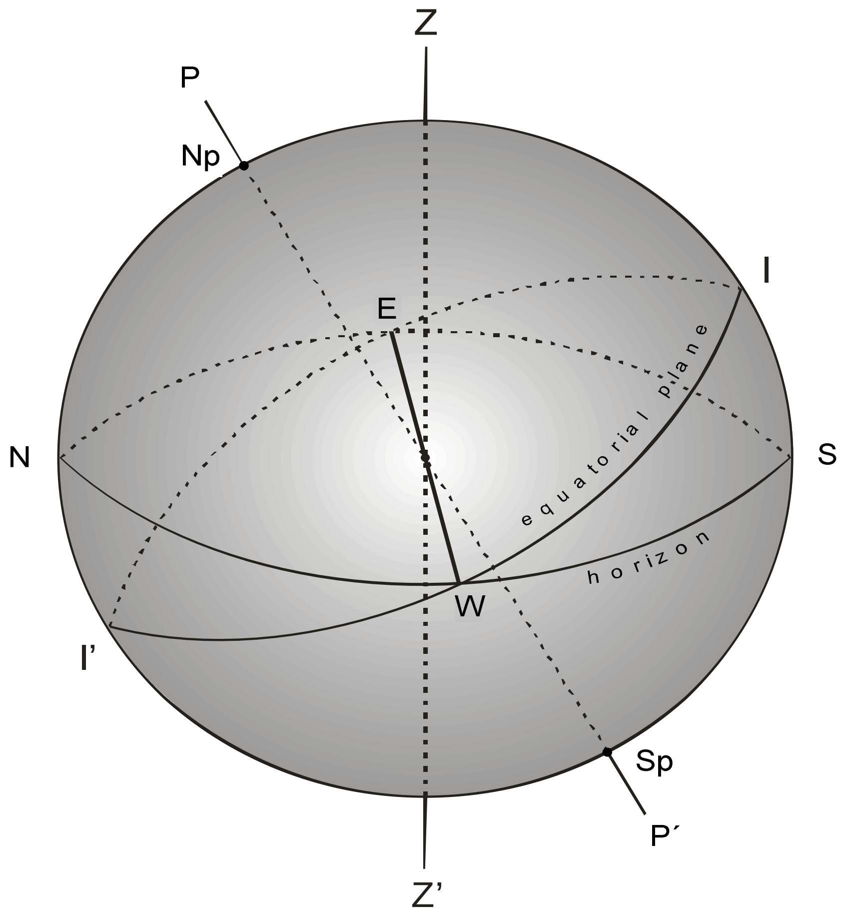

Figure 3.1 Horizon and equatorial plane

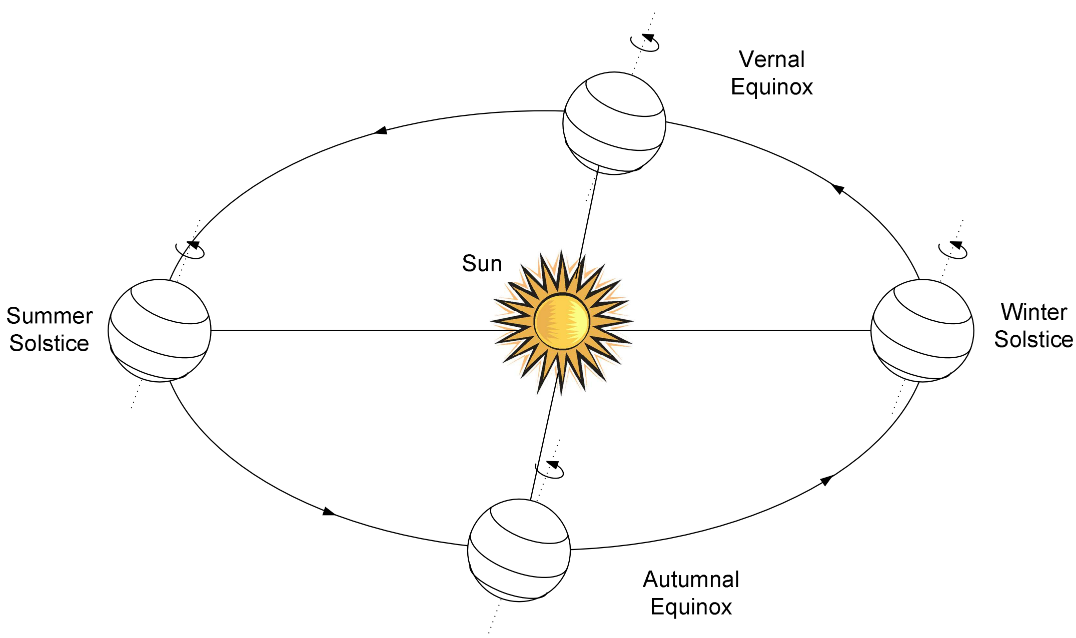

Figure 3.2 Rotation of the earth around its axis and its elliptical orbit around the sun

The plane containing the earth's elliptical orbit is called the ecliptic plane. The seasons are due to the fact that the earth’s axis, which is perpendicular to the equatorial plane, is inclined with respect to the ecliptic plane (Figure. 3.3).

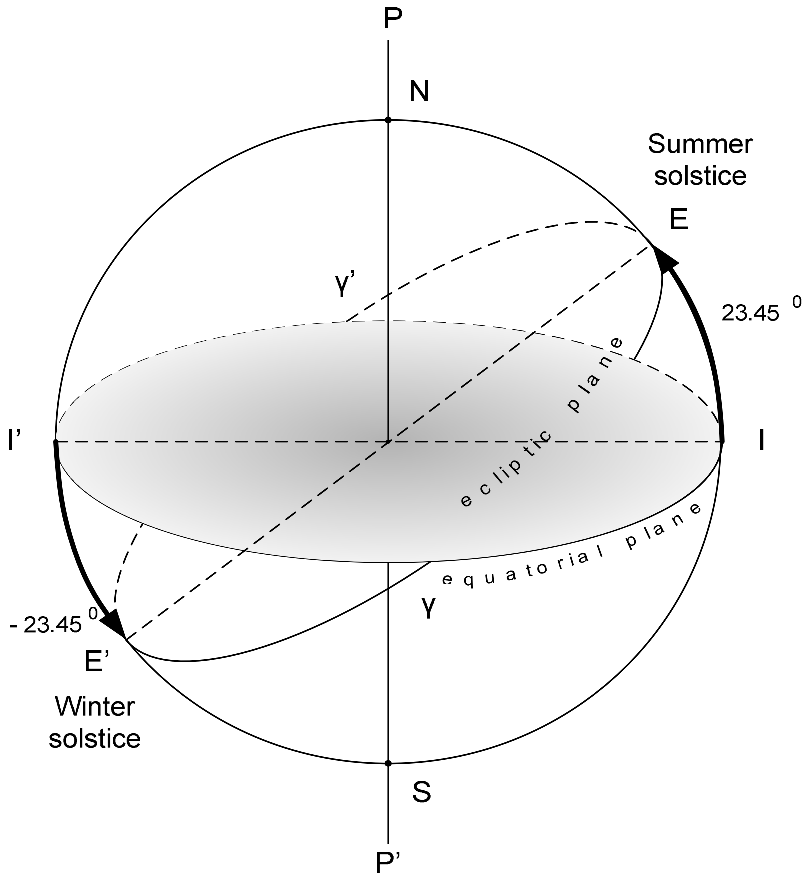

Figure 3.3 Equatorial and ecliptic plane

Also, ecliptic can be defined as the apparent path of the sun’s motion on the celestial sphere as seen from earth. The points γγ′ the ecliptic plane intersects the equatorial plane of the celestial sphere are called equinoxes. These points determine the dates that have 12 hours each of daylight and dark. The most northern excursion of the sun is called the summer solstice and will have the longest amount of daylight. The winter solstice opposite is the shortest period of daylight.

The solar radiation received at different latitudes and in different seasons varies because the axis of rotation of the earth is not perpendicular to the ecliptic plane, but inclined at a fixed angle of 23.45° (Fig. 3.3). Thus, the solar radiation strikes the earth’s Northern hemisphere more directly near the summer solstice, explaining the summer in that hemisphere during that period of the year. At the same time, solar radiation is striking the earth’s southern hemisphere more obliquely, explaining the winter in that region. Figure 3.2 shows that in the summer solstice, the Earth is positioned in its orbit so that the North Pole is tilted 23.45° toward the sun, while in the winter solstice the South Pole is tilted 23.45° toward the sun. During autumn and vernal equinox neither Pole is tilted toward the sun.

3.2 Longitude and Latitude

The geographic coordinate system is a coordinate system that enables every location on the Earth to be specified via two angles, the longitude (L) and the latitude (ϕ). The reference plane is the equatorial plane that is perpendicular to the rotation axis and intersecting the surface of the Earth along the equator. Circles intersecting Earth's surface parallel to the equator determine the **latitude**. The latitude is defined as the angle between the equatorial plane and a line from the earth's centre and a location (T) on the surface of the earth. By definition the latitude is positive in the northern hemisphere and negative in the southern hemisphere.
For the determination of the longitude, one needs a plane perpendicular to the equator including the rotational axis. This plane will create a circle of intersection, or two half circles going from one pole to the other named meridians.
The zero longitude is by definition the meridian passing through Greenwich, UK. The longitude of any location is determined by the angle between the zero meridian and the meridian passing through the location, with positive values for locations west of Greenwich and negative values for locations east of Greenwich. Sometimes, the West/East suffix is used after the value of longitude. These definitions are illustrated in Fig. 3.4.

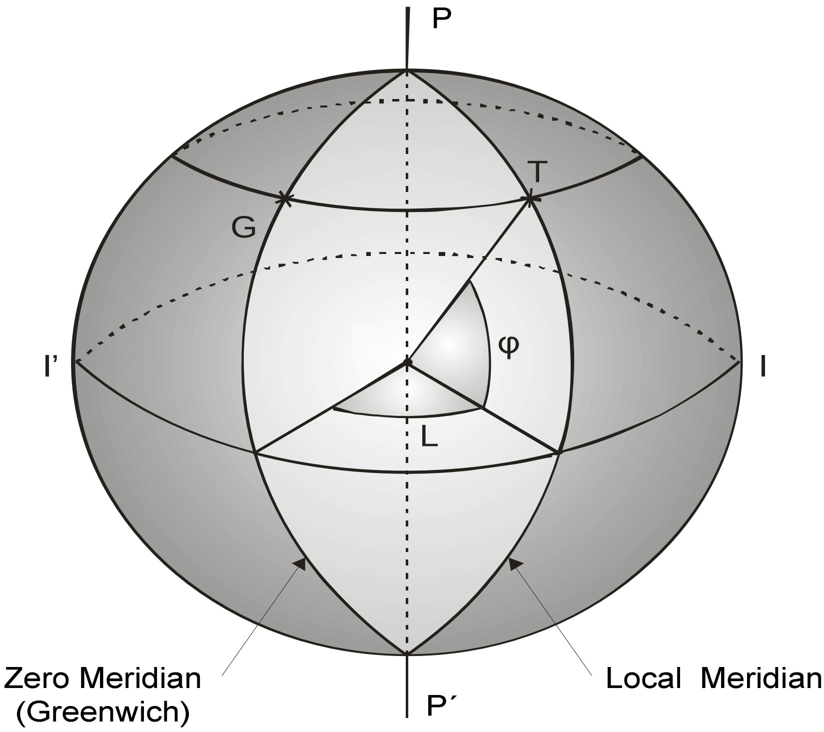

Figure 3.4 Latitude ϕ and longitude L for a location T.

3.3 Basic solar geometry angles

In order to determine the basic solar geometry angles, a basic assumption is that the stars, including the sun, are attached on the surface of the celestial sphere. This imaginary sphere has an arbitrary radius and its centre depends on the different co-ordinate systems. The radius is at sufficiently large distance from the earth, so the location of the sun and other stars can be seen as single points. The centre of the sphere coincides with the position of the observer in the **horizontal system** while in the **equatorial system** coincides with the centre of the earth. Depending on the system, the position of a point moving on the surface of the celestial sphere can be specified if two angles are known. This simple model helps in understanding the diurnal and annual apparent motions as illustrated in Figure 3.5.

In the **horizontal system,** the reference plane is the horizon of the observer. This plane intersects the celestial sphere in the horizon. The intersection of the normal on this plane and the celestial sphere is called the *Zenith* (Z). In this co-ordinate system the position of the sun in the celestial sphere is determined by two angles, the *solar altitude angle* (h) and the *solar azimuth angle* (α).

Solar altitude is the angle made by the line joining the centres of the sun and the observer with its projection along the horizontal plane ranging from -90° to +90°. The solar altitude is positive when the Sun is above the horizon, negative when the Sun is below it. The distance to the Zenith is the complement of the altitude angle and is called *zenith angle* (θ_Z), which is given by

$$\theta_Z = 90° - h \qquad (3.1)$$

The solar azimuth (α) is the angular distance between due south and the horizontal projection of the sun's rays. A positive solar azimuth angle indicates a position west of south and a negative azimuth angle indicates east of south. It is measured from due south in the Northern Hemisphere and from due north in the southern hemisphere. Solar altitude and azimuth angles can be calculated from simple spherical trigonometry equations [1].

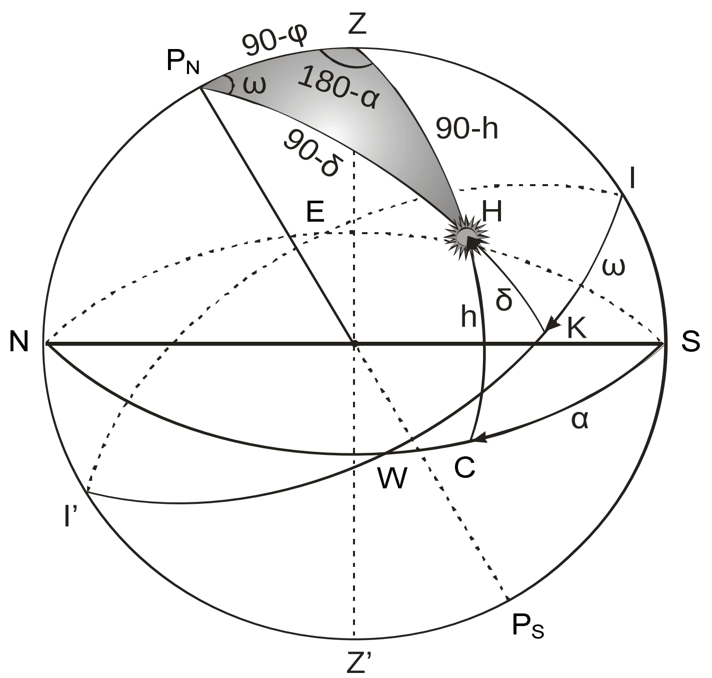

Figure 3.5 The celestial sphere

In the **equatorial system** the reference plane is the equator. The two angles for the determination of the position of the sun on the celestial sphere at any time are the *solar declination angle* (δ) and the *hour angle* (ω).

Solar declination is the angle between the rays of the sun and the plane of the earth's equator. It varies by an angle of up to ± 23° 27' (±23.45°).
This variation causes the changing seasons, with their unequal period of daylight and darkness. The solar declination reaches its maximum value, (+23.45°) in June 21. This day is called summer solstice in the northern hemisphere and winter solstice in the southern hemisphere. During the summer solstice, all locations north of the equator have day lengths greater than twelve hours, while all locations south of the equator have day lengths less than twelve hours. The minimum value, (-23.45°) is reached in December 20. This day marks the winter solstice in the northern hemisphere and summer solstice in the southern hemisphere. During the winter solstice, all locations north of the equator have day lengths less than twelve hours, while all locations south of the equator have day lengths greater than twelve hours. The declination is zero at the vernal equinox (March 21) and at the autumnal equinox (September 22). During these days, the day lengths, regardless of latitude, are exactly 12 hours.

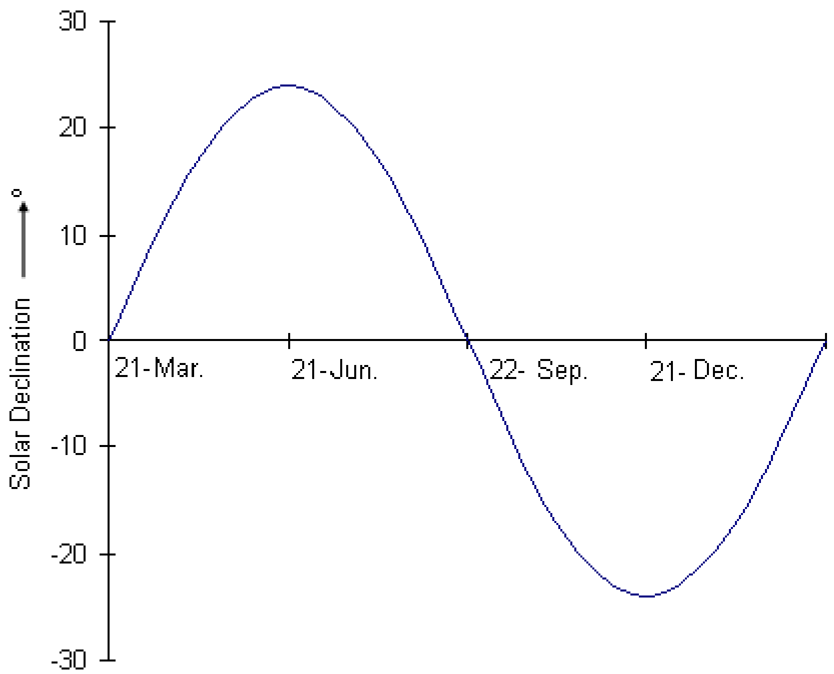

Figure 3.6 Solar declination angle for a year

The declination can be assumed to be constant (as it changes ~ 0.5 deg or less) over a period of day. In Fig. 3.6 this angle is shown in degrees for four characteristic days of the year.

The declination, in degrees, for any given day may be approximately calculated by the following equation [2] :

$$\delta = 23.45 \sin\left[360\left(\frac{284+n}{365}\right)\right] \qquad (3.2)$$

where n is the day of the year.

The solar declination depends on the day of the year. However, if monthly mean values are required, the recommended day of the month for which the declination is closest to the monthly averaged declination has to be used (Table 3.1).

Table 3.1 Recommended average day of each month and corresponding day of the year, with monthly mean value.

Month	Date	Day of the year	Solar declination
Jan.	17	17	-20.9
Feb.	16	47	-13.0
Mar.	16	75	-2.4
Apr.	15	105	9.4
May	15	135	18.8
June	11	162	23.1
July	17	198	21.2
Aug.	16	228	13.5
Sep.	15	258	2.2
Oct.	15	288	-9.6
Nov.	14	318	-18.9
Dec.	10	344	-23.0

Solar hour angle (ω) is the angular distance between the hour circle of the sun and the local's meridian. To an observer on earth the sun appears to move around the earth by the rate of 360^0 in 24 h or 15° per hour. The hour angle is defined as zero at local solar noon, the later being the time of day when the sun's altitude angle is at its greatest. The hour angle decreases by 15^0 for each hour before local solar noon and increases by 15^0 for each hour after solar noon. In other words the hour angle is set as positive after solar noon and negative before solar noon. In calculating the hour angle it is important to use solar time (section 3.5) and not clock time.

Example 3.1
At Athens-Greece (ϕ = 37° 58′), what is the solar declination on February 15?

Solution
On February 15, n = 46 and from equation 3.2:

$$\delta = 23.45 \sin\left(360 \frac{284+46}{365}\right) = -13.29^0$$

Example 3.2
Calculate the solar hour angle at 09.00 and 13.00 solar time.

Solution
According to the definition of hour angle its value at 09.00 will be :

$$\omega = 15 \cdot (9-12) = -45^0$$

Also, at 13.00 solar time we have:

$$\omega = 15 \cdot (13-12) = 15^0$$

3.3.1 Determination of solar angles

The astronomical spherical triangle P_NZH, relative to the earth's coordinates is shown shaded in Fig., 3.5 where sides are labelled.

The law of cosines, for this spherical triangle gives the following expression for the cosine of zenith angle.

$$\cos(90 - h) = \cos(90 - \delta)\cos(90 - \phi) + \sin(90 - \delta)\sin(90 - \phi)\cos\omega \qquad (3.3)$$

or

$$\sin h = \sin\delta \sin\phi + \cos\delta\cos\phi\cos\omega \qquad (3.4)$$

By definition of θ_z (equation (3.1)) the above can be written

$$\cos\theta_Z = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega \qquad (3.5)$$

At solar noon, in the Northern Hemisphere the hour angle (ω) is equal to zero and equation (3.4) gives

$$\sin h_{max} = \cos(\phi - \delta) \qquad (3.6)$$

or

$$h_{max} = 90 - |\phi - \delta| \qquad (3.7)$$

The sunrise and sunset hour angles (ω_s), for horizontal surface, are given by setting the solar altitude angle h of equation (3.4) equal to zero.

$$\cos\phi\cos\delta\cos\omega_s = -\sin\phi\sin\delta \qquad (3.8)$$

$$\cos\omega_s = -\tan\phi\tan\delta \qquad (3.9)$$

Solving equation (3.9) for ω_s , if $-1 \le -\tan\varphi\tan\delta \le +1$ gives

$$\omega_s = \pm \arccos(-\tan\phi\tan\delta) \qquad (3.10)$$

a positive sign corresponds to sunset and a negative sign to sunrise.

In the case where (- $\tan\varphi\tan\delta$) < -1 then the sun never sets on that day (Polar summer), and when (- $\tan\varphi\tan\delta$) > 1 then the sun never rises on that day (Polar winter). Also, in the case where (- $\tan\varphi\tan\delta$) = ±1 then the sun is on the horizon for an instant only.
The day length in hours is given by:

$$\text{Day length} = \frac{2\omega_s}{15} \tag{3.11}$$

By applying the rule of sinus to the spherical triangle P_NZH, we have

$$\frac{\sin(90 - h)}{\sin\omega} = \frac{\sin(90 - \delta)}{\sin(180 - \alpha)} \tag{3.12}$$

or

$$\frac{\cosh}{\sin\omega} = \frac{\cos\delta}{\sin\alpha} \tag{3.13}$$

or

$$\sin\alpha = \frac{\cos\delta\sin\omega}{\cosh} \tag{3.14}$$

The azimuths of the sunrise and sunset may be obtained from equation (3.14) by setting the solar altitude *h* to zero and by substituting the values of ω_s to ω we have:

$$\cos\alpha = -\frac{\sin\delta}{\cos\phi} \tag{3.15}$$

In this case there are two solutions for the morning and two for the afternoon. The correct solutions will be selected such that when the declination is positive the sun rises and sets north of the east-west line, and when is zero it rises due east and sets due west, and when is negative, it rises and sets south of the east-west line.

An application of the apparent sun paths for Athens (Greece), is given graphically directly below.

Figure 3.7 shows the apparent sun's path across the sky for Athens (Greece) and the relative location of sunrise and sunset, for winter solstice (Fig. 3.7[a]), equinoxes (Fig. 3.7[b]), and summer solstice (3.7[c]). Also the solar altitude angle at solar noon is indicated in each Figure. It should be noted that, during the equinoxes the sun rises due east and sets due west as shown in Figure 3.7[b], and the day length is equal to the night length. Between the vernal and autumnal equinoxes the sun rises and sets to the north of east - west line, the day length is longer and at solar noon of the summer solstice the solar altitude reaches its maximum value (Figure. 3.7[c]). Between the autumnal and vernal equinox the sun rises and sets to the south of east - west line, the day length is shorter and at solar noon of the winter solstice the solar altitude reaches its minimum value throughout the year (Figure. 3.7[a]).

For the graphical representation of the apparent motion of the sun on the celestial vault, for each day of the year, and for a given location, a software program can be used [3].

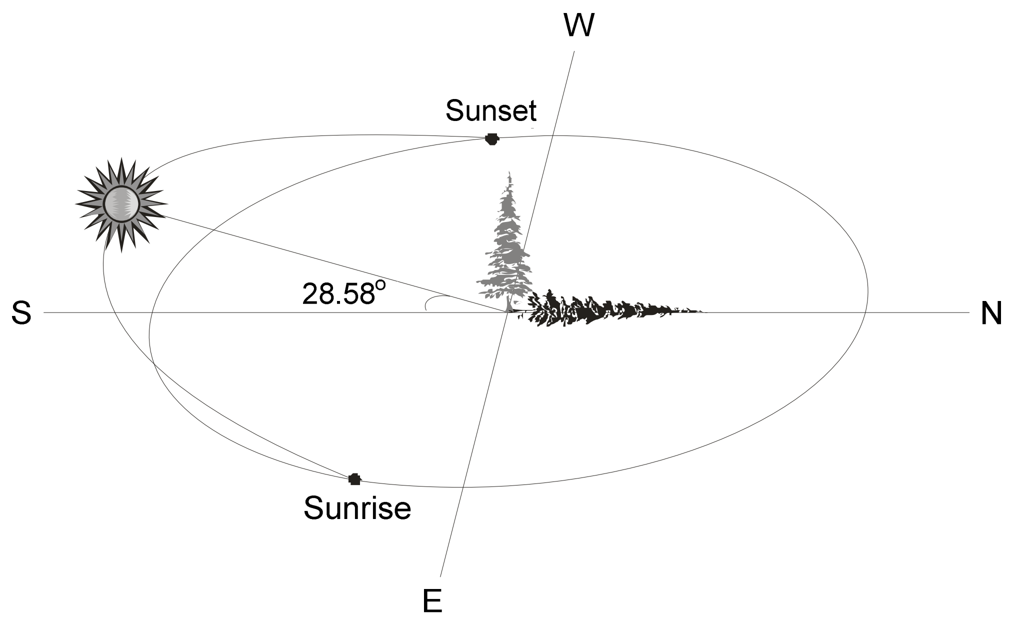

Figure 3.7[a] Apparent sun's path for winter solstice (Athens- Greece)

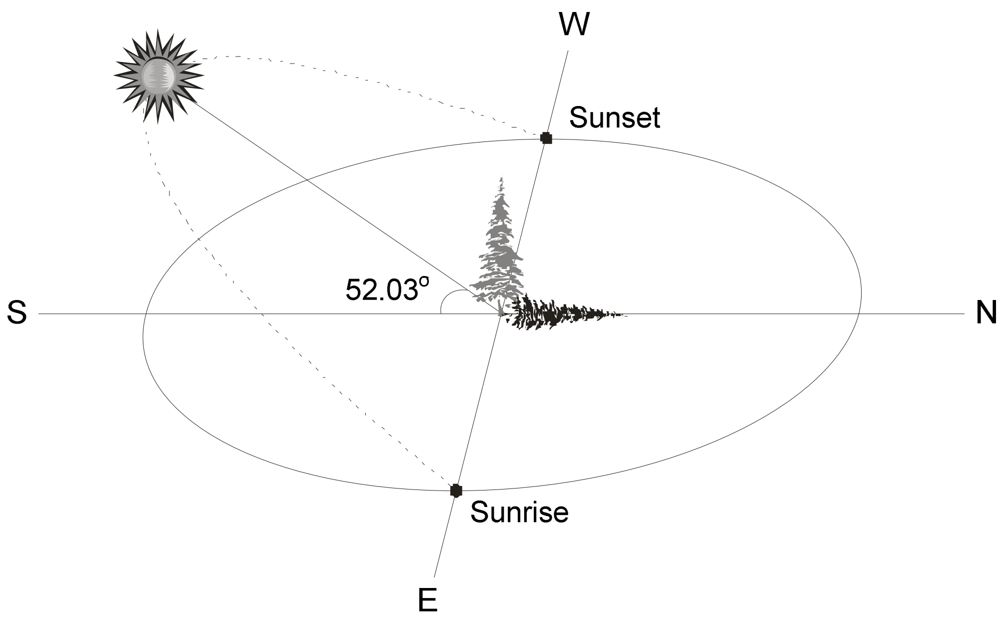

Figure 3.7[b] Apparent sun's path for equinoxes (Athens- Greece)

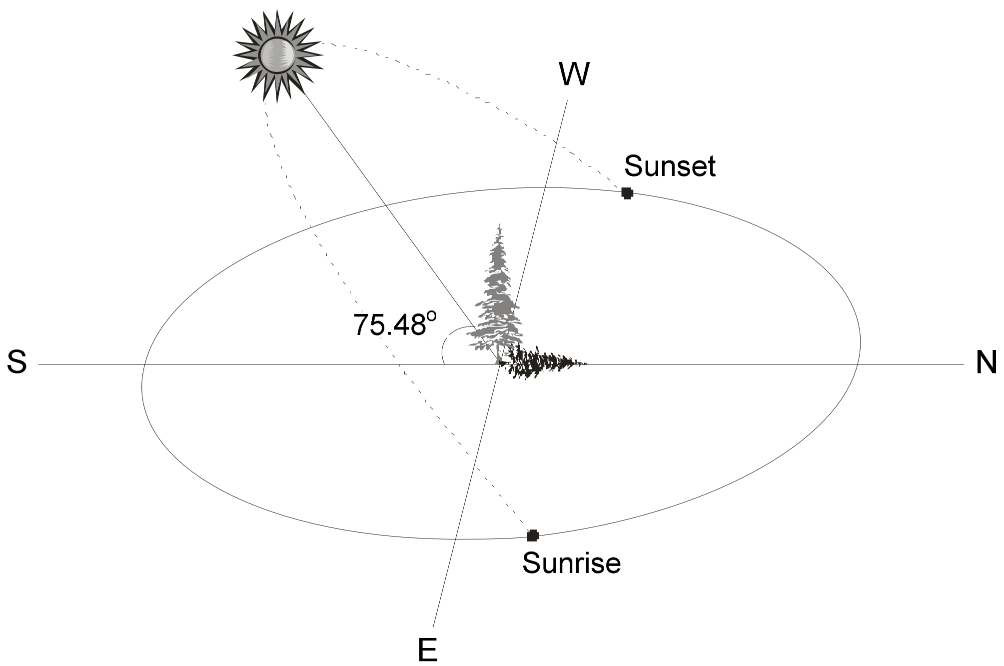

Figure 3.7[c] Apparent sun's path for summer solstice (Athens- Greece)

Example 3.3

Calculate the solar azimuth angle at Athens-Greece ($\phi = 37^0\ 58'$), on February 25 at

14.00 solar time.

Solution

On February 25, n = 56 and from equation 3.2:

$$\delta = 23.45 \sin\left(360\frac{284+56}{365}\right) = -9.78^0$$

The hour angle ω, is:

$$\omega = 15\cdot(14-12) = 30^0$$

From equation 3.4 the solar altitude angle (h) is :

sinh = sin(-9.78)sin37.97+cos(-9.78)cos39.97cos30 = 0.57

or h = 34.63^0

The solar azimuth angle can be calculated with equation 3.14:

$$\sin a = \frac{\cos(-9.78)\sin 30}{\cos 34.63} = 0.60$$

or α = 36.79^0

Example 3.4

Calculate the zenith angle of the sun at Athens-Greece (ϕ = 37° 58′) at solar noon on July 20.

Solution

On July 20, n = 201 and from equation 3.2:

$$\delta = 23.45 \sin\left(360\frac{284+201}{365}\right) = 20.64^0$$

At solar noon the solar altitude angle is given by equation 3.7:

$$h_{max} = 90\text{-}(37.97\text{-}20.64) = 72.67^0$$

Thus the zenith angle of the sun at solar noon is given by equation 3.1:

$$\vartheta_Z = 90-72.67 = 17.33^0$$

3.4 Solar Incidence Angle

The solar incidence angle is very useful, as it allows a relatively simple calculation of the radiation incident on a surface. The angular relationships between the direct solar radiation incident on a plane, such as a wall surface or glazing area, oriented arbitrarily relative to the earth can be described in terms of several angles. These angles are illustrated in Figure 3.8. The orientation and the tilt of the surface is determined with two angles respectively: the *surface azimuth angle* (γ) and the *slope* (β).

Surface azimuth angle is the angle between the south and the projection of the surface normal in the horizontal plane. This angle is taken positive if the normal is west of south and negative if east of south.

Slope is the angle at which the surface is inclined from horizontal and is taken positive for south-facing surfaces.

Also, the position of the sun relative to the surface can be expressed using the *solar incident angle* (θ).

Solar incident angle is the angle between the surface's normal and the sun's rays.
The interrelationships of the previous defined angles can be calculated from simple spherical trigonometry equations, applying the rule of cosines to the spherical triangle HKC. The angle of incidence of beam radiation on a surface can be related by a general equation to the slope of the surface, the solar zenith angle, the solar azimuth angle and the surface azimuth angle. In Figure 3.8, KO is the normal to horizontal surface, CO is the normal to inclined surface and OZ is the horizontal projection of the normal to inclined surface and applying the rule of cosines to the spherical triangle HKC, we have

$$\cos\theta = \cos\beta\cos\theta_Z + \sin\beta\sin\theta_z\cos(\alpha-\gamma) \tag{3.16}$$

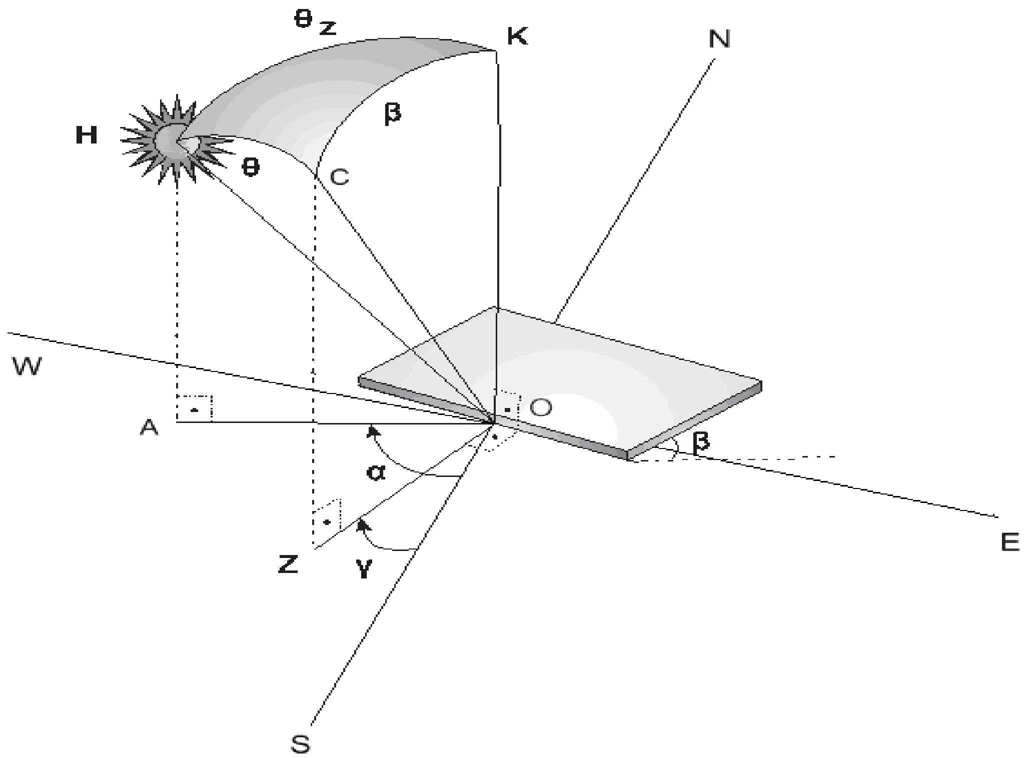

Figure 3.8 Various sun – earth angles for a tilted surface.

In the case of an inclined surface, the hour angle at sunrise or sunset ω_{st}, may be less

than the value given by equation (3.10), if the corresponding incidence angle is greater than 90°. For such a situation the hour angle ω_{st} can be found by modifying equation (3.16), taking into account the equations (3.5) and (3.14).
Therefore, the incidence angle in terms of the latitude, slope, declination and hour angle is given by:

$$\begin{aligned}\cos\theta &= \sin\delta(\sin\phi\cos\beta - \cos\phi\sin\beta\cos\gamma) \\ &+ \cos\delta\cos\omega(\cos\phi\cos\beta + \sin\phi\sin\beta\cos\gamma) + \cos\delta\sin\beta\sin\gamma\sin\omega\end{aligned} \tag{3.17}$$

If the surface is facing the equator (γ = 0), equation (3.17) gives:

$$\cos\theta = \sin\delta(\sin\phi\cos\beta - \cos\phi\sin\beta) + \cos\delta\cos\omega(\cos\phi\cos\beta + \sin\phi\sin\beta) \tag{3.18}$$

or

$$\cos\theta = \sin(\phi-\beta)\sin\delta + \cos(\phi-\beta)\cos\delta\cos\omega \tag{3.19}$$

Comparing the above equation (3.19) to the solar zenith angle of equation (3.5), results that a south – facing surface has an effective latitude of (ϕ-β).
Finally, the sunrise or sunset hour angles ω_{st} for an inclined surface are obtained by putting θ = 90° in equation (3.19) and solving for ω.

$$\omega_{st} = \pm\arccos(-\tan(\phi-\beta)\tan\delta) \tag{3.20}$$

However, since ω_{st} cannot have values greater than ω_s, the complete equation for ω_{st} is given by:

$$\omega_{st}' = \min\{\omega_s, \arccos(-\tan(\phi-\beta)\tan\delta)\} \tag{3.21}$$

The sunrise (ω'_{sr}) and sunset (ω'_{ss}) angles for an inclined surface that does not face due south, will not be symmetrical about solar noon and may be obtained from equation 3.17 by setting the incidence angle θ = 90°. This solution gives two values for ω depending on the surface orientation [4].

For γ<0

$$\omega'_{sr} = -\min\left\{\omega_{s,}\ \cos^{-1}\left[\frac{a\cdot b - \sqrt{(a^2 - b^2 + 1)}}{a^2+1}\right]\right\} \tag{3.22}$$

$$\omega'_{ss} = \min\left\{\omega_{s,}\ \cos^{-1}\left[\frac{a\cdot b + \sqrt{(a^2 - b^2 + 1)}}{a^2+1}\right]\right\} \tag{3.23}$$

For γ>0

$$\omega'_{sr} = -\min\left\{\omega_{s,}\ \cos^{-1}\left[\frac{a\cdot b+\sqrt{\left(a^2-b^2+1\right)}}{a^2+1}\right]\right\} \quad (3.24)$$

$$\omega'_{ss} = \min\left\{\omega_{s,}\ \cos^{-1}\left[\frac{a\cdot b-\sqrt{\left(a^2-b^2+1\right)}}{a^2+1}\right]\right\} \quad (3.25)$$

where:

$$a = \frac{\cos\phi}{\sin\gamma\tan\beta} + \frac{\sin\phi}{\tan\gamma} \quad (3.26)$$

$$b = \tan\delta\left(\frac{\cos\phi}{\tan\gamma} - \frac{\sin\phi}{\sin\gamma\tan\beta}\right) \quad (3.27)$$

The “min” in the equations 3.22 – 3.25, means the smaller of the two items in the brackets. Also, for the above equations, it is assumed that the surface azimuth angle is measured from due south positive westward and negative eastward.

All the previous defined angles are useful in solar radiation calculations. The angle of incidence of direct solar radiation determines the intensity of the direct component striking the surface and the ability of the surface to reflect, transmit, or absorb solar radiation. Knowledge of this component is necessary for determining the total solar radiation on inclined surfaces.

Example 3.5

Calculate the angle of incidence for a flat plate solar collector that faces 14° west of south and has a slope of 40^0 in Athens-Greece (ϕ = 37° 58′) on May 20 at 13.00 *h* solar time.

Solution

On May 20, n = 140 and from equation 3.2:

$$\delta = 23.45\sin\left(360\frac{284+140}{365}\right) = 19.93^0$$

The hour angle ω, is:

$$\omega = 15\cdot\left(13-12\right) = 15^0$$

From equation 3.17 we have:

$$\begin{aligned}\cos\theta &= \sin 19.93(\sin 37.97\cos 40 - \cos 37.97\sin 40\cos 14)\\ &+\cos 19.93\cos 15(\cos 37.97\cos 40 + \sin 37.97\sin 40\cos 14)\\ &+\cos 19.93\sin 40\sin 14\sin 15 = 0.943\end{aligned}$$

Hence, the angle of incidence for the solar collector is θ = 19.42°

Example 3.6

A south – facing flat plate solar collector is located in Athens - Greece (ϕ = 37° 58′) with a slope of 45°. Calculate the solar time of sunset for this collector on September 14.

Solution

On September 14, n = 257 and from equation 3.2:

$$\delta = 23.45 \sin\left(360 \frac{284+257}{365}\right) = 2.62^0$$

From equation 3.10, we have:

$$\omega_s = \arccos(-\tan 37.97 \tan 2.62) = 92.05^0$$

and from equation 3.21:

$$\arccos(-\tan(37.97-45)\tan 2.62) = 89.68^0$$

According to equation 3.21 the solar sunset angle on the collector is the smaller of the two calculated values. Hence, the solar sunset angle on the collector is 89.68^0 and the solar time for sunset is at 17.98 h.

Example 3.7

A flat plate solar collector is located in Athens - Greece (ϕ = 37° 58′) with a slope of 60° and facing 20° east of south. Calculate the day length that the sun shines on this collector on October 14.

Solution

On October 14, n = 287 and from equation 3.2:

$$\delta = 23.45 \sin\left(360 \frac{284+287}{365}\right) = -9.23^0$$

Since the collector does not face due south, from equations 3.26 and 3.27 we have:

$$a = \frac{\cos 37.97}{\sin(-20)\tan 60} + \frac{\sin 37.97}{\tan(-20)} = -3.02$$

$$b = \tan(-9.23) \cdot \left(\frac{\cos 37.97}{\tan(-20)} - \frac{\sin 37.97}{\sin(-20)\tan 60}\right) = 0.18$$

Using equation 3.10, we have:

$$\omega_s = \pm \arccos(-\tan 37.97 \tan -9.23) = 82.71^0$$

Since γ<0 the sunrise (ω'_{sr}) and sunset (ω'_{ss}) hour angles for the collector are given by equations 3.22 and 3.23 respectively.

$$\omega'_{sr} = -\min\left\{\omega_s, \cos^{-1}\left[\frac{-3.02\cdot 0.18-\sqrt{\left(3.02^2-0.18^2+1\right)}}{3.02^2+1}\right]\right\} = -\min\left\{82.71^0, 111.56^0\right\}$$

$$\omega'_{ss} = \min\left\{\omega_s, \cos^{-1}\left[\frac{-3.02\cdot 0.18+\sqrt{\left(3.02^2-0.18^2+1\right)}}{3.02^2+1}\right]\right\} = \min\left\{82.71^0, 74.92^0\right\}$$

Thus, the sunrise and sunset hour angles on the inclined collector are - 82.71° and 74.92° respectively. Finally, the day length for the collector is 10.51 hours, while for horizontal surface is 11.03 hours.

3.5 Solar Time

Time specific to the sun does not coincide with local clock time for two reasons. The first is the changes in the rotational and orbital angular speed of the earth. This correction called the equation of time (ET), and can be determined approximately from Figure 3.9 or can be expressed [5] in equation form (in minutes) as:

$$ET = 229.2(0.000075 + 0.001868\cos B - 0.032077\sin B - 0.014615\cos 2B - 0.04089\sin 2B) \quad (3.28)$$

where $B = (n-1)\frac{360}{365}$ and n is the day of the year.

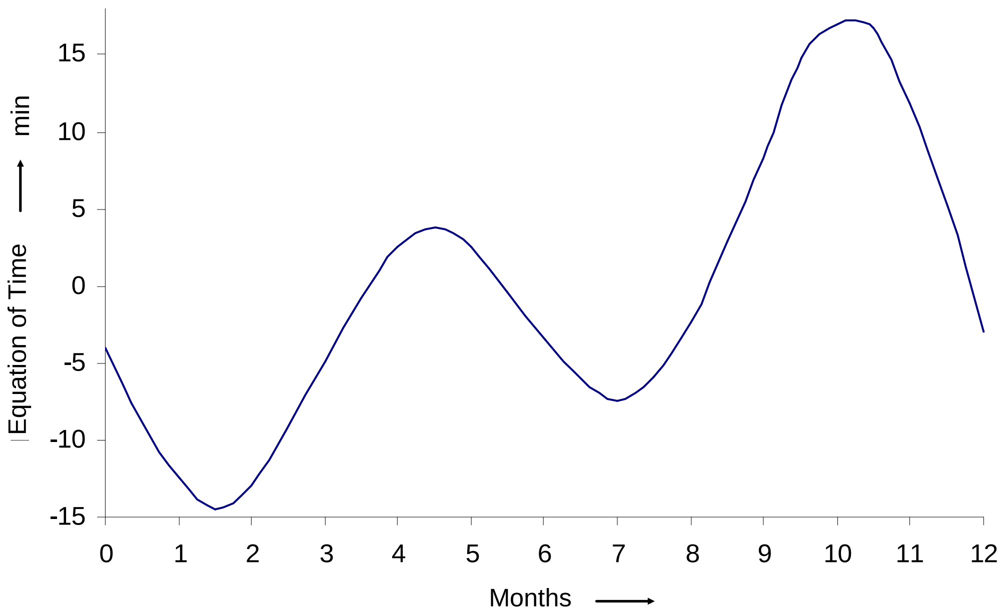

Figure 3.9 The equation of time (ET) as a function of time of year

The second is the difference in longitude between the location (L_{loc}) and the meridian with which the standard time is based (L_{st}). This correction has a magnitude of 4 minutes for every degree difference in longitude. In many countries, clocks are advanced one hour beyond Standard Time in summer; such time is called Daylight Savings Time (C). Thus, the solar time can be written as:

$$\text{Solar Time} = \text{Standard Time} + ET \pm 4(L_{st} - L_{loc}) + C \qquad (3.29)$$

where L_{st} is the standard meridian for the local time zone, L_{loc} is the longitude of the location and ET is the equation of time [6]. A local time zone is a region of the earth that has adopted the same standard time. Standard time zones can be defined by geometrically subdividing the surface of the Earth into 24 zones, bordered by meridians each 15° of longitude in width. For example, the standard meridian for Italy is 15° and for Greece is 30°. Daylight Savings Time correction C is equal to zero if not on Daylight Savings Time, otherwise C is equal to the number of hours that the time is advanced for Daylight Savings Time, usually 1hr.

Note that all the terms in the above equation must be converted to minutes. The positive sign in the third term of this equation is for places West of Greenwich while the negative sign is for places East of Greenwich.

Example 3.8

Calculate a) the local time of sunset in Athens-Greece (ϕ = 37° 58′, L= 23° 43′) on April 19 if C = 0 and b) the day length for this day.

Solution

a) On April 19, n = 109 and from equation 3.2:

$$\delta = 23.45 \sin\left(360 \frac{284+109}{365}\right) = 10.9^0$$

The sunrise and sunset hour angles (ω_s), are given by equation 3.10:

$$\omega_s = \arccos(-\tan 37.97 \tan 10.9) = 98.64^0 = 6.58\,h$$

Therefore, sunset is at 6.58 h solar time.

From equation 3.28 the time correction on April 19 is 0.765 min. Therefore, rearranging equation 3.29 we have:

Local standard time = 394.8 – 0.765 + 4(30 - 23.72) = 419.2 min ≈ 7.0 *h*

Hence, sunset is at 19.00 *h* local standard time.

b) From equation 3.11 the day length is:

$$\text{Day length} = \frac{2 \cdot 98.64}{15} = 13.15\ h$$

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